

Goo Ishikawa

**Singularities
of Curves and Surfaces
in Various Geometric Problems**

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Cover design: **Małgorzata Zielińska**

DTP: *Fixpoint*

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Warsaw 2016

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www.csz.pw.edu.pl

ISBN: 978-83-61993-15-5

Printed in Poland

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Introduction

This lecture note provides a basic exposition of the geometric singularity theory by several concrete examples and the classification results of singularities of differentiable mappings, in particular, in situations where various geometric structures come across. Mainly curves and surfaces are treated. They are simple objects but they possess possibility to help and lead to deep understanding why and how singularities appear in geometry. The author wishes to clarify the fundamental objects and methods, and proposes one of the principal prospects on the geometric study of singularities and applications.

Chapters 1 and 2 are devoted to the basic notions and methods of general singularity theory which we need. In Chapters 3 and 4, certain singularities which appear in Euclidean geometry are discussed. Chapter 5 is dedicated to singularities in projective geometry, Chapter 6 to those in contact and symplectic geometry, and chapter 7 to those in conformal geometry, respectively. Several open problems are mentioned in chapter 8. Chapter 9 (Appendix) is for the basic/advanced material throughout the lecture note and for further reading of the referred books and papers.

A naive knowledge of geometry will be enough to read throughout this lecture note. The author actually hopes that reading this lecture text will stimulate a deeper understanding of geometry.

Sections with the mark * contain advanced studies, which can be skipped for the first reading.

The mappings which we treat in this note are assumed to be of class C^∞ unless otherwise stated.

The author would like to express his gratitude to Professor dr. hab. Stanisław Janeczko for giving him the opportunity to write this lecture note and to the office of Center for Advanced Studies, Warsaw University of Technology, for the nice hospitality and pleasant atmosphere.

March 2015 Warsaw
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